Naval Research Laboratory
Stennis Space Center, MS 39529-5004

NRL/MR/7174--96-8018

Acoustic Reverberation from Bubble Plumes in the Ocean: Do Multiple Scattering Processes Play a Significant Role?

CHRISTOPHER FEUILLADE

Ocean Acoustics Branch
Acoustics Division

January 17, 1997

REPORT DOCUMENTATION PAGE

Form Approved
OBM No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	January 17, 1997	Final

4. TITLE AND SUBTITLE

Acoustic Reverberation from Bubble Plumes in the Ocean: Do Multiple Scattering Processes Play a Significant Role?

5. FUNDING NUMBERS

Job Order No. 5717018-00
Program Element No. 0602435N
Project No.
Task No.
Accession No. DN16-3889

6. AUTHOR(S)

Christopher Feuillade

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Research Laboratory
Acoustics Division
Stennis Space Center, MS 39529-5004

8. PERFORMING ORGANIZATION REPORT NUMBER

NRL/MR/7174--96-8018

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Office of Naval Research
800 N. Quincy Street
Arlington, VA 22217-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Acoustic scattering from ocean bubble plumes has recently increased in importance as a cause of sea surface reverberation. With the development and application of high frequency SONAR equipment, the question has arisen as to whether multiple scattering processes between strongly resonating bubbles significantly affects reverberation levels. This report describes a comparison of two sets of experimental data with both the classic theory of linear acoustic propagation in bubbly water, and a recently published new theory which includes multiple scattering effects. The results of the analysis indicate that, for water containing bubbles of uniform size with volume fractions $\beta \geq 0.22\%$, multiple scattering effects strongly influence acoustic behavior. However, when $\beta < 0.22\%$, the effects of multiple scattering appear to diminish quite sharply. Also, for nonuniform bubbles and $\beta \approx 0.02\%$, the analysis shows that multiple scattering effects have no observable effect. The overall implication of these results is that, for realistic compositions of bubbly water found in the ocean, multiple scattering effects do not affect the attenuation and dispersion of sound, and may be neglected. Consequently, the classic theory should be completely adequate to describe acoustic scattering and reverberation levels from ocean bubble plumes.

14. SUBJECT TERMS	15. NUMBER OF PAGES
acoustic propagation, acoustic scattering, bubbles, shallow water	17
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

ACOUSTIC REVERBERATION FROM BUBBLE PLUMES IN THE OCEAN: DO MULTIPLE SCATTERING PROCESSES PLAY A SIGNIFICANT ROLE ?

C. Feuillade

Naval Research Laboratory, Stennis Space Center, Mississippi 39529-5004

ABSTRACT

Acoustic scattering from ocean bubble plumes has recently increased in importance as a cause of sea surface reverberation. With the development and application of high frequency SONAR equipment, the question has arisen as to whether multiple scattering processes between strongly resonating bubbles significantly affects reverberation levels. This report describes a comparison of two sets of experimental data with both the classic theory of linear acoustic propagation in bubbly water, and a recently published new theory which includes multiple scattering effects. The results of the analysis indicate that, for water containing bubbles of uniform size with volume fractions $\beta \geq 0.22\%$, multiple scattering effects strongly influence acoustic behavior. However, when $\beta < 0.22\%$, the effects of multiple scattering appear to diminish quite sharply. Also, for nonuniform bubbles and $\beta \approx 0.02\%$, the analysis shows that multiple scattering effects have no observable effect. The overall implication of these results is that, for realistic compositions of bubbly water found in the ocean, multiple scattering effects do not affect the attenuation and dispersion of sound, and may be neglected. Consequently, the classic theory should be completely adequate to describe acoustic scattering and reverberation levels from ocean bubble plumes.

INTRODUCTION

Acoustic scattering from ocean bubble plumes has recently attained a high degree of importance and recognition as a primary cause of sea surface reverberation.[1] Scattering from bubble plumes is due to a change in the acoustic impedance of the medium through which sound propagates. The presence of air bubbles changes the compressibility of water within a plume, such that an external sound field will encounter it as an object with acoustic properties different from the ambient fluid. As a result, some of the incident acoustic energy is scattered from the boundary of the plume.

The pioneering research in this area was performed by Carstensen and Foldy.[2] They introduced the concept of the "effective" medium, by which bubbly water is treated as a homogeneous fluid with uniform acoustic properties (i.e., attenuation coefficient and phase speed). The theory they developed (termed here the "classic" theory) has been widely and successfully used for many years, although typically at frequencies well below the individual bubble resonance. It allows a bubble plume to be treated as a single "collective" scattering object with representative acoustic parameters. The collective scattering nature of bubble plumes at low frequencies has recently received a convincing experimental demonstration.[3] The reader should see refs. 4, 5 for a helpful background discussion of this subject, and an extensive list of references.

When the frequency of ensonification is such that the monopole resonance of the bubbles is strongly excited, the classic theory is known to give inaccurate descriptions of the properties of bubbly water. This is especially true when the bubbles are densely populated and of uniform size.[6] In a recent paper, Feuillade[7] showed that the principal reason for the failure of the classic theory at frequencies close to the bubble resonance is that it does not incorporate multiple scattering processes between bubbles. By correctly including these processes, a new theory was developed which accurately represented the properties of bubbly water in the resonance region.

Individual air bubbles in water have remarkable acoustical properties. At their monopole resonance frequencies they may have scattering cross-sections many times larger than their physical size. With the development and application of SONAR equipment operating at frequencies high enough to excite the monopole resonances of bubbles found at the ocean surface, the question has arisen as to whether multiple scattering processes between strongly resonating bubbles significantly affects the scattering behavior and, consequently, the levels of reverberation from bubble plumes. One approach to answering this question (adopted in this present study) is to determine if the classic theory, which omits multiple scattering effects, gives an accurate description of the collective properties of bubbly water at resonance frequencies (as evidenced by the acoustic attenuation), for the compositions of bubble sizes and volume fractions typically encoutered in ocean bubble

plumes. If it does, then the classic theory may be confidently used to predict the scattering behavior of bubble plumes, and multiple scattering effects may be safely neglected.

The generation and evolution of ocean bubble plumes has been investigated, among others, by Monahan.[8] He describes that they are generally produced by spilling waves, and pass through several stages. At the instant of production, the air volume fraction (β) of plumes may be as high as 8%, but this initial phase is of very short duration (~1s). In the second phase (lasting 3.5–4.3s), the volume fraction β is typically 0.01–0.2%. In the third phase (lasting 10^2–10^3s), and subsequent development, β is 0.0001% or less. At all stages, plumes contain a wide spectrum of bubble sizes, with radii in the range 0.02mm–0.4mm.

Two data sets are available, one obtained by Silberman,[6] and the other by Fox et al.,[9] which exhibit the experimentally measured acoustic properties of bubbly water for volume fractions over the range 0.02–1%. These experimental results thus cover the high end of the acoustically significant second phase of ocean bubble plume evolution. By comparing them with the classic theory,[2] and also with the recently published new theory of acoustic propagation in bubbly water which includes multiple scattering effects,[7] it is possible to determine the significance of multiple scattering effects for reverberation from ocean plumes.

The data analysis results presented here show that, for water containing bubbles of uniform size with volume fractions $\beta \geq 0.22\%$, multiple scattering effects strongly influence acoustic behavior. However, when $\beta < 0.22\%$, the effects of multiple scattering appear to diminish quite sharply. As detailed in Monahan's work, bubbles typically found in ocean plumes are not uniform, but have a broad spectrum of radii. This analysis shows that, for nonuniform bubbles and $\beta \approx 0.02\%$, multiple scattering effects have no observable effect. The overall implication of these results is that, for realistic compositions of bubbly water found in the ocean, multiple scattering effects do not affect the attenuation and dispersion of sound, and may be neglected. Consequently, the classic theory should be completely adequate to describe acoustic scattering and reverberation levels from ocean bubble plumes.

THE SPEED OF SOUND IN BUBBLY WATER

According to the classic theory of acoustic propagation in bubbly water,[2,10a,11] the sound speed c_T in water containing f bubbles per unit volume, all with the same radius a, is given by :

$$\frac{1}{c_T^2} = \frac{1}{c^2} + \frac{4\pi a f}{\omega^2\left\{\left[\frac{\omega_0^2}{\omega^2} - 1\right] + i\,\delta\right\}} \quad . \tag{1}$$

In this expression, c is the sound speed in pure water, ω_0 is the angular resonance frequency and ω represents the frequency of ensonification. The quantity δ is the damping constant for the bubble, which has radiative, thermal and viscous components, i.e.,

$$\delta = \delta_r + \delta_t + \delta_v \quad . \tag{2}$$

The exact mathematical forms, and typical values, of the components δ_r, δ_t and δ_v for air bubbles in water, used here, are described in ref. 10b. The sound speed in bubbly water is seen to be a complex quantity. Its real and imaginary parts represent the dispersion and attenuation, respectively, of acoustic waves propagating through the medium.

If bubbles of different sizes are present in the water, then the number of bubbles per unit volume with radius between a and a+da is defined as fp(a)da, where p(a) is a PDF, and (1) becomes :

$$\frac{1}{c_T^2} = \frac{1}{c^2} + \int_0^\infty \frac{4\pi a f p(a)\, da}{\omega^2 \left\{ \left[\frac{\omega_0^2}{\omega^2} - 1 \right] + i\,\delta \right\}} \quad . \tag{3}$$

(It should be noted that δ is also a function of the bubble radius a.) Both expressions (1) and (3) do not include the effects of multiple acoustic scattering between bubbles. Feuillade[7] showed how these processes may be included in the analysis to replace (1) by :

$$\frac{1}{c_T^2} = \frac{1}{c^2} + \frac{4\pi a f}{\omega^2 \left\{ \left[\frac{\omega_0^2}{\omega^2} - 1 \right] + i\,\delta - 4\pi a f \int_0^\infty r\, e^{-i\kappa r}\, dr \right\}} \quad . \tag{4}$$

Comparison of (4) with (1) shows that an additional term containing an integral appears in the denominator. This integral represents the aggregate multiple scattering effect of all the other bubbles surrounding any particular bubble in the medium. The new term is generally a complex quantity, whose real and imaginary parts lower the resonance frequency and increase the damping, respectively, of the individual bubbles. The quantity κ, appearing in the integrand, is a complex wavenumber representing propagation in the bubbly water. Expression (4) reduces to (1) when the bubble density $f \rightarrow 0$.

The expression for the sound speed in water containing bubbles of nonuniform radius is likewise modified when multiple scattering processes are included. Expression (3) becomes :

$$\frac{1}{c_T^2} = \frac{1}{c^2} + \frac{\displaystyle\int_0^\infty \frac{4\pi a f\, p(a)\, da}{\omega^2\left\{\left[\frac{\omega_0^2}{\omega^2} - 1\right] + i\,\delta\right\}}}{1 - \displaystyle\int_0^\infty r\, e^{-ikr}\, dr \int_0^\infty \frac{4\pi a f\, p(a)\, da}{\left\{\left[\frac{\omega_0^2}{\omega^2} - 1\right] + i\,\delta\right\}}} \quad . \tag{5}$$

Comparison of (5) with (3) shows that a new term has again appeared in the denominator, representing the multiple scattering effect of surrounding bubbles. We see that (5) reduces to (3) when the bubble density is low (i.e., $f \rightarrow 0$). If all the bubbles are of equal radius, then (5) reduces to (4).

ANALYSIS OF EXPERIMENTAL DATA

To determine the importance of multiple scattering processes in bubbly water, two experimental data sets are analyzed: that of Silberman[6] ; and that of Fox, Curley and Larson.[9] In these comparisons, the necessary physical parameters for water and air are derived either from the published experimental report (if recorded) or from the CRC Handbook of Chemistry and Physics.[12]

The classic theory of propagation in bubbly water leads to (1) and (3), from which the complex sound speed c_T may be calculated. Similarly, in the new theory, which includes multiple scattering effects, c_T is derived either from (4) or (5). From each of these equations, the corresponding propagational attenuation coefficient A may be determined by the following procedure (see page 735 of ref. 13 for further details). Let the relevant equation [i.e., either (1), (3), (4) or (5)] be manipulated to yield a complex expression for the quantity (c / c_T), which may be formally written

$$\frac{c}{c_T} = u - i\,v \quad , \tag{6}$$

where u and v indicate the real and imaginary components respectively of (c / c_T). The attenuation coefficient A, in dB per unit length, is then given by

$$A = 20 \log_{10}(e) \left(\frac{\omega v}{c}\right) \approx 8.68589 \left(\frac{\omega v}{c}\right) . \qquad (7)$$

Following this procedure, values for A are compared against experimental data in the following examples.

(a) Silberman[6]

The details of Silberman's experimental procedure are fully explained in ref. 6, and also summarized on page 736 of ref. 13. Great care was taken in the control of bubble size in this experiment. Air volume fractions $0.025\% < \beta \leq 1\%$ were used, and ensembles of bubbles with a high degree of size uniformity produced. Various runs were performed, with bubble radii varying between about 1mm (resonance frequency ≈ 3.27kHz) and 3.5mm (resonance frequency ≈ 933Hz).

Figure 1 shows the attenuation coefficient as a function of frequency (note the logarithmic scales on the axes) for one of the intermediate values of the volume fraction studied by Silberman, i.e., $\beta = 0.22\%$. The data points were obtained during different runs for bubbles of radii a = 1.77mm, 1.83mm, 2.07mm, and 2.44mm. The solid curve (curve 1) shows the theoretical attenuation coefficient A predicted by the classic theory (1), via (7), for bubbles of single uniform radius a = 1.77mm (resonance frequency = 1.84kHz). Comparison of curve 1 with data points for bubbles of the same radius (denoted "◇") at frequencies close to the resonance peak shows, when all the bubbles in the medium have the same radius, that the classic theory overestimates the attenuation coefficient in the resonance region. Below the resonance frequency, however, the classic theory fits the data very well, even for bubbles of different radius. As noted earlier, the classic theory has been used very successfully to describe low frequency reverberation from ocean bubble plumes.

The classic theory omits radiative interactions between bubbles. The new theory,[7] which includes radiative interactions, may be implemented to make predictions which agree with the data near resonance, by replacing κ with the wavenumber k for pure water in the integral in (4), and evaluating the integral to some finite upper limit $R \approx r_0$ (rather than ∞), where r_0 is the average distance between adjacent bubbles in the medium. The value of R may be interpreted as the radius of a sphere circumscribed around any bubble encompassing those neighbors with which the bubble has strong multiple scattering interactions. The inclusion of multiple scattering interactions has the effect of reducing the attenuation peak and shifting it to a lower frequency. In figure 1, the dashed curve 2 indicates the values of A obtained via this procedure, and was fitted to the data by adjusting

R so as to minimize the least squares difference between the experimental points for 1.77mm bubbles ("◇"), and the corresponding points on the curve for the same set of frequencies. Since the individual bubble resonances are very narrow in Silberman's experiment (the resonance region itself is actually contained within a narrow strip centered upon the steep side of the curves in figure 1), the frequency downshift due to multiple scattering has an anomalously large effect on the errors near the actual resonance, and the least squares fitting has been restricted to the 1.77mm data points on the plateau above the resonance frequency. In this case, the optimal value of $R = 2.79$cm. Curve 2 also continues to fit well to the experimental points for bubbles of different radius below the resonance frequency. If the volume of the circumscribing sphere (of radius $R = 2.79$cm) is multiplied by f (for $\beta = 0.22\%$), then the number of strongly interacting neighboring bubbles is given by $\mathscr{N} (= 4\pi f R^3/3) = 8.64$, indicating that the reduced level of attenuation exhibited by the resonance data points in figure 1 will be observed if every bubble in the medium interacts strongly, on average, with between eight and nine surrounding bubbles. Since $r_0/a \propto \beta^{-1/3}$ (for spherical bubbles of all radii), the effects of multiple scattering in water with different volume fractions may be usefully compared with each other by scaling the average separation between adjacent bubbles r_0 in units of the bubble radius. For this case the average value of $r_0/a = 12.39$. Similarly, $R/a = 15.76$, so that $R/r_0 = 1.272$ (i.e., the radius of the region of strong multiple interaction is of the same order as, but slightly greater than, the average bubble separation, which agrees well with the assumption made to perform the integral).

Silberman performed several experimental runs with volume fractions $\beta > 0.22\%$, and several with $\beta < 0.22\%$. These experimental data were processed in a manner identical to that just described for the $\beta = 0.22\%$ case. The following table presents all the results obtained from analysis of Silberman's acoustic attenuation data for water containing uniform distributions of bubbles.

β	Bubble radius, a	r_0/a	R/a	R/r_0	$\mathscr{N}$
1.0%	2.59mm	7.48	9.59	1.282	8.80
0.53%	2.07mm	9.25	11.89	1.285	8.87
0.22%	1.77mm	12.39	15.76	1.272	8.64
0.0584%	1.92mm	19.29	14.17	0.735	1.66
0.0377%	0.994mm	22.31	19.34	0.867	2.73

What is immediately noticeable from this table is the consistency of the results for the higher volume fraction data, which indicate that when $1\% \geq \beta \geq 0.22\%$, every bubble in the medium has strong multiple scattering interactions with about $\mathscr{N}$ = 8–9 of its neighbors. Similarly, the radius of the spherical region around each bubble containing these neighbors (scaled in units of r_0) also remains quite constant, with $R/r_0 \approx 1.28$. These interactions dominate the acoustic response of the medium in the resonance region and will strongly affect the propagation of sound (and, therefore, the scattering effect of bubbly water). The results for the lower volume fraction cases appear to show that when $0.22\% > \beta > 0.0584\%–0.0377\%$, the number of strongly interacting neighbors falls to about 1.5–3, indicating a sharp diminishment in the level of multiple scattering between bubbles somewhere within this range. Since $r_0/a \propto \beta^{-1/3}$, radiative interactions should decrease even more when β is further reduced. This means that, somewhat below a volume fraction β = 0.0377%, the acoustic properties will be determined by the independent behavior of the bubbles. Correspondingly, the scattering effect of bubbly water should be well described by the classic theory when the volume fraction falls below this value.

(b) Fox, Curley and Larson[9]

Analysis of Silberman's data shows that multiple scattering can have an significant effect on the acoustic properties of bubbly water for the higher volume fractions levels typically encoutered in the ocean. His data, however, are for cases where all the bubbles in the medium have practically the same radius. This condition never occurs in the ocean. Monahan[8] indicates that bubble plumes contain a broad spectrum of bubble radii. An opportunity to study the effects of multiple scattering in bubbly water with a broad distribution of bubble sizes, and for volume fraction levels found in ocean bubble plumes, is provided by the data of Fox, Curley and Larson.[9] The details of this experiment are explained in ref. 9, and summarized on pp. 740-741 of ref. 13. Water containing bubbles with a broad spectrum of radii was produced by blowing air through a fine filter. The resonance frequencies were higher than in Silberman's work due to the use of smaller bubbles, with a typical radius around 0.04mm. The bubble radius distribution is described by the histogram of volume fractions shown in figure 2. This histogram was produced by measuring the diameters of over 1500 bubbles, and should be used only to determine the *relative* volume of air in each radius category. It will be noted from figure 2 that the bubble radii vary between 0.01mm and about 0.12mm, which covers the lower 25% of the range found in ocean bubble plumes, as quoted by Monahan. During the experimental runs, Fox et al. measured the total volume of air in bubbles of all categories, and estimated it to vary between $2\pm0.5 \times 10^{-4}$ cc of air per cc of water (i.e., a volume fraction $\beta = 0.02\pm0.005\%$). For the calculations reported here, each of the ten radius

categories was divided into five strips, and values of f corresponding to the mean bubble radius and volume fraction in each strip were calculated.

Fox et al. performed numerous experimental runs over a period of time. Their collected data (from all runs) show considerable variability, indicating that the total volume fraction was probably slightly different for each run. Fortunately, they also present clear attenuation data taken during a single run, which are much less variable. These are analyzed here. For this data set it is helpful, for the purposes of comparison, to use a measure of the "goodness of fit" between theory and experiment. This is obtained by calculating the "sum-squared error" (denoted ε) between the various curves and the data points.

Figure 3 shows the attenuation coefficient as a function of frequency (note the linear scales on the axes). The specific value of the total volume fraction for this run was not recorded. Curve 1 shows the theoretical attenuation coefficient A predicted by the classic theory for water containing bubbles of nonuniform radius (i.e., Eq. 3). The calculation incorporated a bubble radius distribution f(a) derived from figure 2, taking the total volume fraction as the mean value within the range quoted by Fox et al. (i.e., $\beta = 0.02\%$). We see that the classic theory represents the general shape of the frequency distribution of the data quite well, but overestimates the level of the attenuation coefficient in the resonance region. The sum-squared error between curve 1 and the attenuation data points is $\varepsilon = 5.10 \times 10^3$. Curve 2 represents the values of A predicted by the new theory from (5) where, as before, κ was replaced with the wavenumber k for pure water, and the integral over r was evaluated to a finite upper limit R. The value of R used (= 0.189cm) was obtained by least squares fitting to all the data points, which produced a minimum value for the sum-squared error, i.e., $\varepsilon_{min} = 2.89 \times 10^3$. The corresponding value of $\mathscr{N} = 9.68$, a number which includes bubbles of various sizes. The peak of curve 2 is reduced, which is expected. However, since the volume fraction distribution shown in figure 2 represents a much greater population of small bubbles than large bubbles, multiple scattering effects cause the high frequency components for the more abundant small bubbles to be shifted and reduced relatively more than the low frequency components. As a result, curve 2 also becomes *asymmetrical* in overall shape. Note that this behavior is not suggested by the data points, which show no indication of either a frequency shift or asymmetry. In fact, a much more convincing fit to the data can be achieved with the classic theory (i.e., without including any multiple scattering effects) by simply *reducing* the value of the total volume fraction in the calculation. Curve 3 in figure 3 shows the attenuation coefficient produced by (3), using $\beta = 0.0118\%$, where β was treated as a variable parameter and determined by least squares fitting to the data points. For curve 3, $\varepsilon_{min} = 5.76 \times 10^2$. Comparison of the data points with curve 3 suggests a constant offset between theory and experiment in the resonance region. The value of $\beta = 0.0118\%$ is about 1.64 standard deviations below the quoted mean value

of $\beta = 0.02\%$, and a systematic error may have been incurred in measuring very small bubble radii. Below resonance all the theoretical curves fit the data well, as seen earlier with the Silberman data. This result strongly suggests that the classic theory is adequate to explain the experimental results of Fox et al., implying that multiple scattering effects do not play a significant role in the propagation of sound in this medium.

Why should multiple scattering be less important with a range of bubble radii present? The reason is that individual bubble resonances are narrow, and radiative interactions are most effective in the case of closely spaced bubbles resonating at the same frequency.[14] Since a broad spectrum of bubble radii is present in this experiment, it is unlikely that close neighbors will have the same, or even overlapping, resonances. The bubbles resonate independently of each other, and the acoustic behavior is well represented by an integral over the individual *uncoupled* resonances, as incorporated in (3). If multiple scattering effects do not play a significant role in the propagation of sound through water containing bubbles with radii between 0.01mm and 0.12mm, and with $\beta \approx 0.02\%$, then it seems quite reasonable to expect that this will also be true for the wider bubble distributions found in ocean bubble plumes (i.e., radii between 0.02mm and 0.4mm), for similar values of β.

CONCLUSIONS

At the ocean surface, the air volume fraction within bubble plumes, during their most acoustically significant phase, typically ranges from $\beta = 0.01\%$ up to $\beta = 0.2\%$. Analysis of the experimental data of Silberman shows that multiple scattering processes do affect the acoustical properties of bubbly water when $\beta = 0.22\%$. However, in these experiments, the water contained bubbles which all had the same radius. These bubbles therefore all resonated at the same frequency, and would be expected to interact strongly with each other at the resonance frequency. Further analysis of Silberman's data showed that when the volume fraction was reduced below $\beta = 0.22\%$, the degree of multiple scattering interactions diminished sharply, even though the bubbles were still uniformly sized.

The bubbly water found in ocean plumes contains a very broad spectrum of bubble sizes. Analysis of the data obtained by Fox, Curley and Larson, in which $\beta \approx 0.02\%$ and the bubbles had a broad range of radii (although not as broad as found in ocean bubble plumes), implies that multiple scattering should play no observable role in the propagation of sound through water containing bubbles of nonuniform radius, at this level of the volume fraction. It seems that the influence of multiple scattering is probably insignificant with a broad range of radii, where neighboring bubbles are likely to resonate at different frequencies.

Overall, the results of the data analysis reported here strongly imply that, for typical distributions of bubbles found at the ocean surface, multiple scattering effects are insignificant. The classic theory of acoustic propagation in bubbly water should, as a consequence, be completely adequate to describe the acoustic reverberation properties of ocean bubble plumes.

REFERENCES

1 F.S. Henyey,"Acoustic scattering from ocean microbubble plumes in the 100 Hz to 2 kHz region," J. Acoust. Soc. Am. **90**, 399-405 (1991); B.E. McDonald,"Echoes from vertically stratified subresonant bubble clouds: A model for ocean surface reverberation," J. Acoust. Soc. Am. **89**, 617-622 (1991); P.M. Ogden and F.T. Erskine,"Surface and volume scattering measurements using broadband explosive charges in the Critical Sea Test 7 Experiment," J. Acoust. Soc. Am. **96**, 2908-2920 (1994); D.M. Farmer and S. Vagle, "Waveguide propagation of ambient sound in the ocean-surface bubble layer," J. Acoust. Soc. Am. **86**, 1897-1908 (1989); A. Prosperetti, N.Q. Lu and H.S. Kim,"Active and passive acoustic behavior of bubble clouds at the ocean's surface," J. Acoust. Soc. Am., **93**, 3117-3127 (1993).

2 E.L. Carstensen and L.L. Foldy, "Propagation of sound through a liquid containing bubbles," J. Acoust. Soc. Am. **19**, 481-501 (1947).

3 M. Nicholas, R.A. Roy, L.A. Crum, H. Oğuz and A. Prosperetti, "Sound emissions by a laboratory bubble cloud," J. Acoust. Soc. Am. **95**, 3171-3182 (1994).

4 *Physics of sound in the sea,* (U.S. Government Printing Office, Washington DC, 1946, reprinted by Peninsula Publishing, Los Altos, CA), Ch. 28.

5 T.G. Leighton, *The Acoustic Bubble* (Academic Press, London, 1994), Ch 3, §3.8.

6 E. Silberman, "Sound velocity and attenuation in bubbly mixtures measured in standing wave tubes," J. Acoust. Soc. Am. **29**, 925-933 (1957).

7 C. Feuillade, "The attenuation and dispersion of sound in water containing multiply-interacting air bubbles," J.Acoust.Soc.Am., **99**, 3412-3430 (1996).

8 E.C. Monahan, "Occurrence and evolution of acoustically relevant sub-surface bubble plumes and their associated, remotely monitorable, surface whitecaps," in *Natural Physical Sources of Underwater Sound,* edited by B.R. Kerman (Kluwer Academic Publishers, Netherlands, 1993), pp. 503-516.

9 F.E. Fox, S.R. Curley and G.S. Larson, "Phase velocity and absorption measurements in water containing air bubbles," J. Acoust. Soc. Am. **27**, 534-539 (1955).

10 C. S. Clay and H. Medwin, *Acoustical Oceanography* (John Wiley, New York, 1977): (a) pp. 469-472 ; (b) pp. 465-467.

11 P.M. Morse and K.U. Ingard, *Theoretical Acoustics* (McGraw-Hill, New York, 1968), pp. 407-414.

12 *CRC Handbook of Chemistry and Physics* (CRC Press, Boca Raton FA, 1983), 64th ed.

13 K.W. Commander and A. Prosperetti, "Linear pressure waves in bubbly liquids: Comparison between theory and experiments," J. Acoust. Soc. Am. **85**, 732-746 (1989).

14 C. Feuillade, "Scattering from collective modes of air bubbles in water and the physical mechanism of superresonances," J.Acoust.Soc.Am., **98**, 1178-1190 (1995).

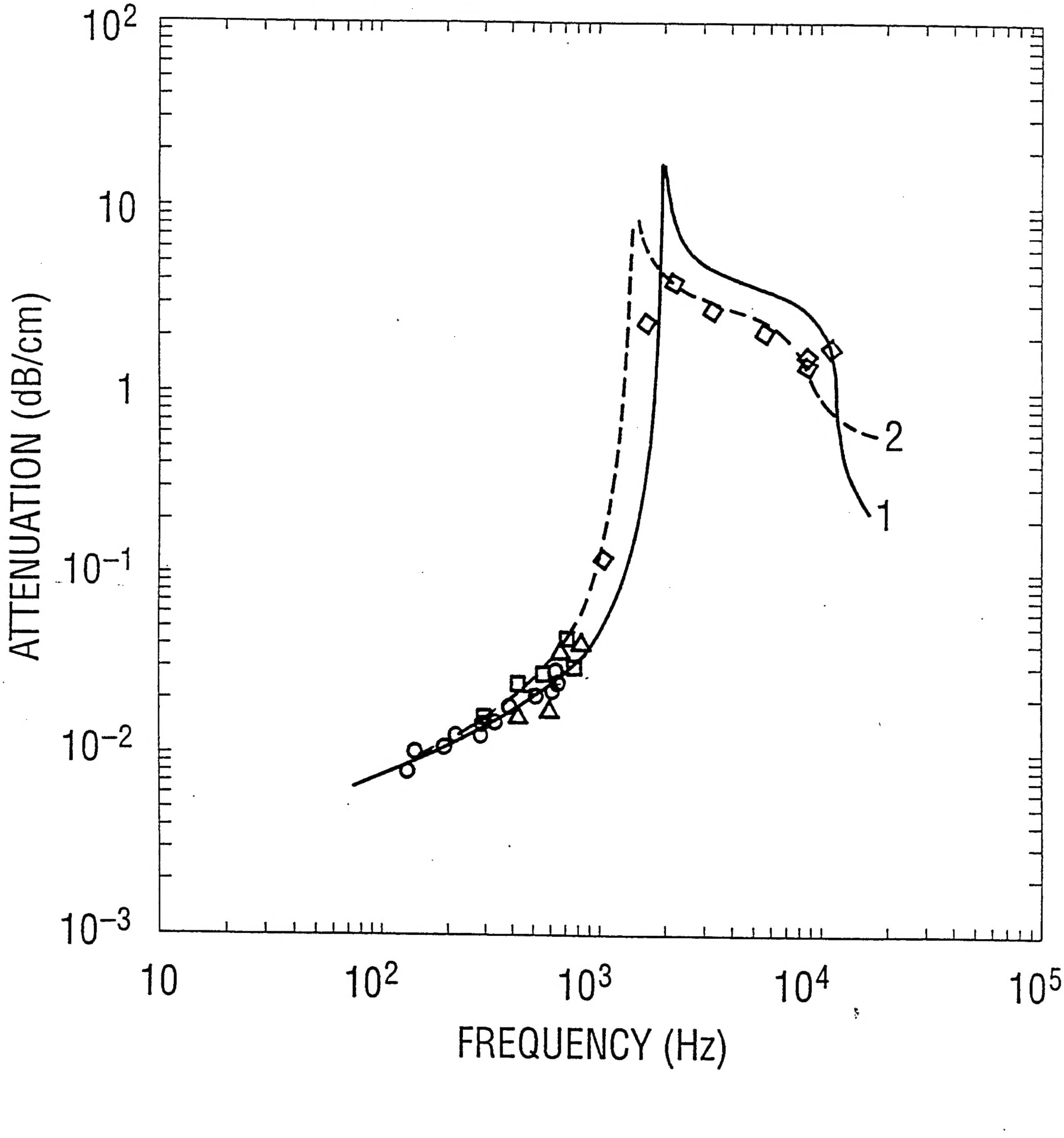
Figure Captions

Fig. 1 Attenuation coefficient for $\beta = 0.22\%$, from Silberman's data (ref. 6). Note the logarithmic scales on the axes. The data points are for bubbles of radius: 1.77mm ("◇"); 1.83mm ("○"); 2.07mm ("□"); 2.44mm ("△"). Curve 1 shows the attenuation coefficient predicted by the classic theory (eq. 1) for bubbles of radius 1.77mm. Curve 2 shows the attenuation coefficient predicted by the new theory (eq. 4) for 1.77mm bubbles, but by evaluating the integral with upper limit $R = 2.79$cm. This value of R was determined by least squares fitting to the 1.77mm points above the resonance frequency.

Fig. 2 Bubble distribution in experiment of Fox, Curley and Larson. This histogram shows the relative volume fraction for bubbles falling into a broad range of radius categories, produced by measuring the sizes of over 1500 bubbles. The overall distribution must be scaled to give the total air volume fraction measured during the experiment.

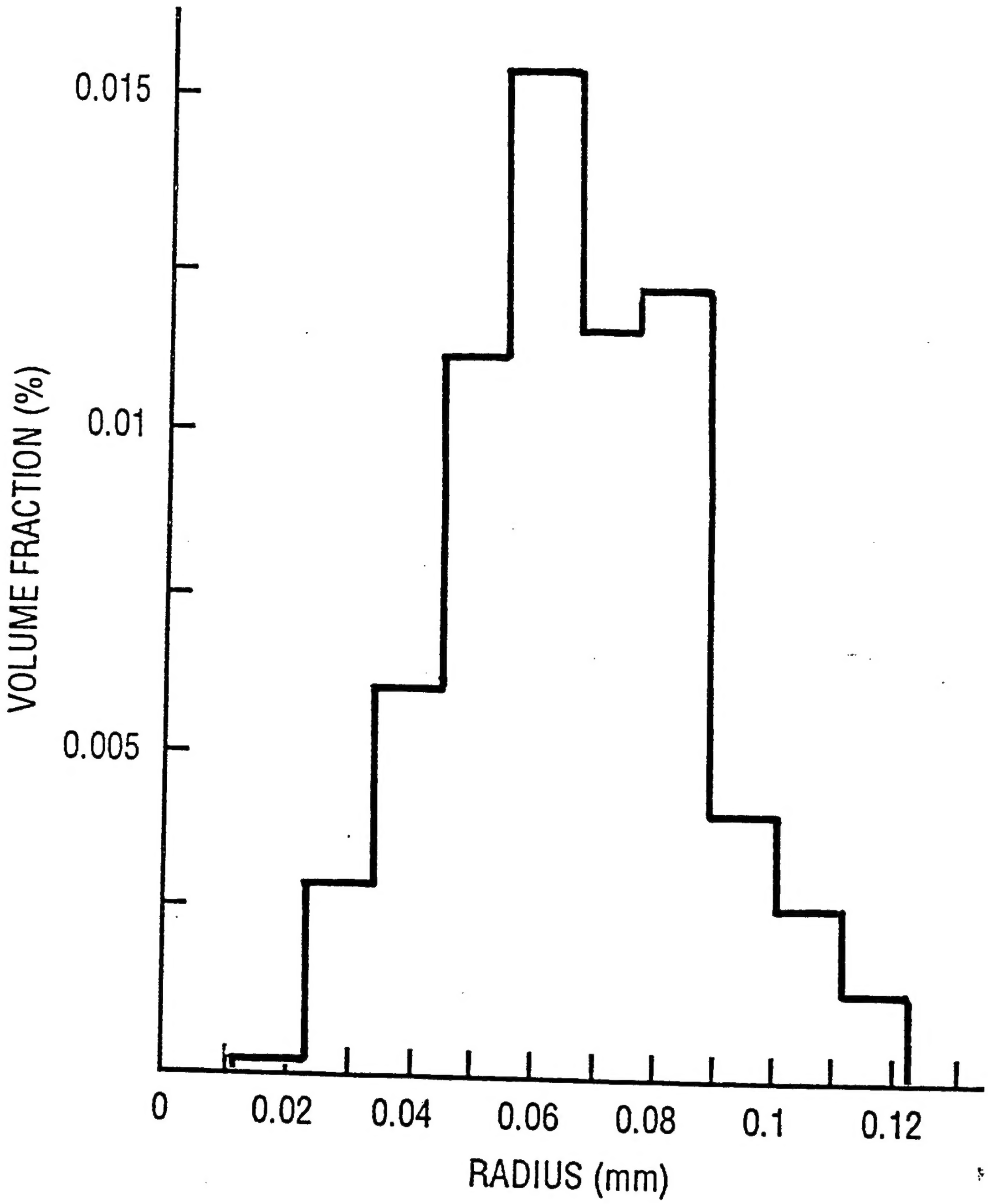
Fig. 3 Attenuation coefficient for water containing a broad range of bubble sizes. Note the linear scales on the axes. The data points ("•") were taken by Fox, Curley and Larson (ref. 9). Curve 1 shows the attenuation coefficient predicted by the classic theory for water containing differently sized bubbles (eq. 3), assuming $\beta = 0.02\%$. Curve 2 shows the attenuation coefficient predicted by the new theory (eq. 5) for the same volume fraction, but by evaluating the integral with upper limit $R = 0.189$cm. This value of R was determined by least squares fitting to all the points. Curve 3 shows the attenuation coefficient predicted by the classic theory (eq. 3), using $\beta = 0.0118\%$. This value of β was obtained by least squares fitting to the points.

FIG 1



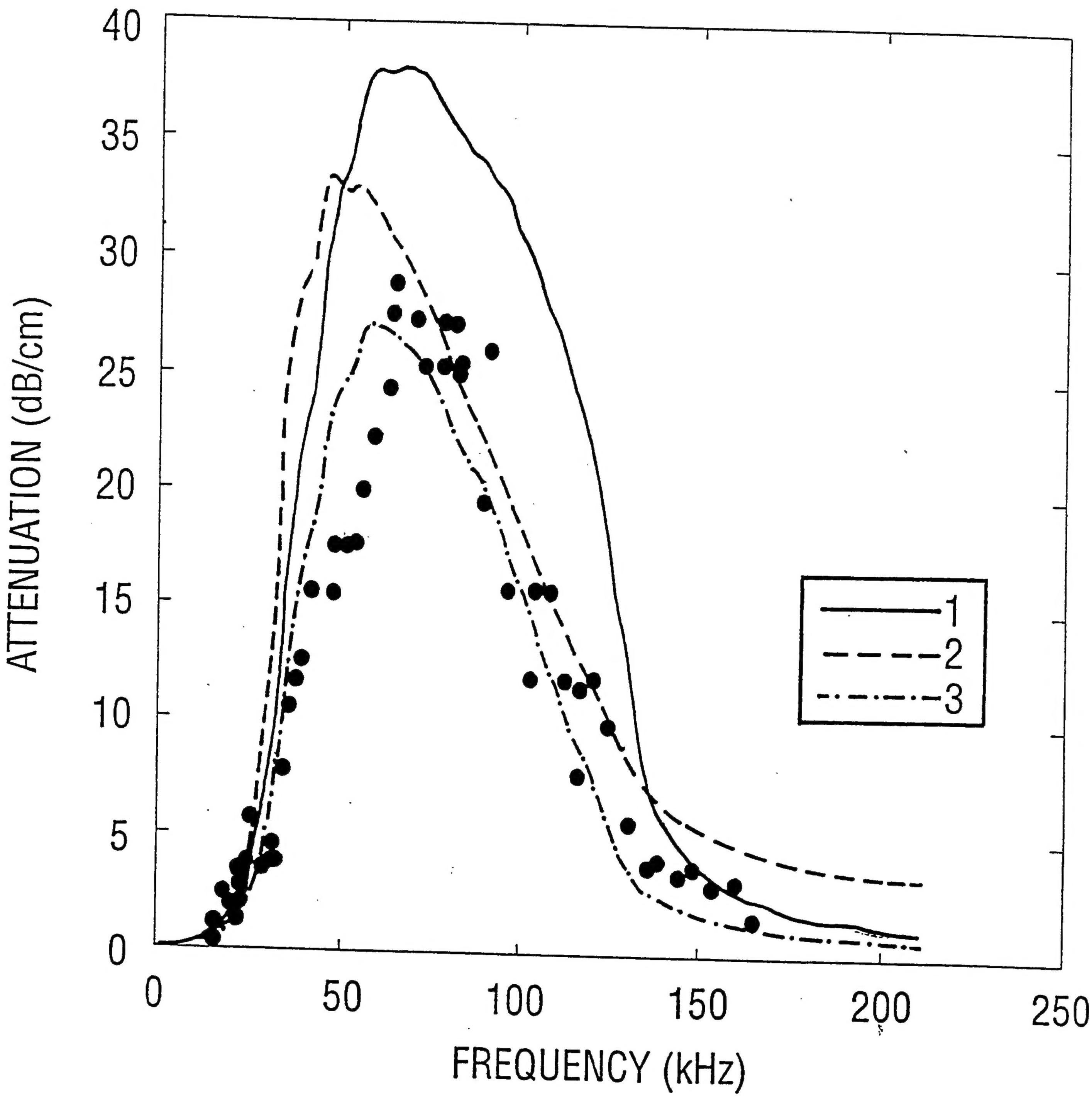
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
ATTENUATION (dB/cm)
2
1
10
10^2
10^3
10^4
10^5
FREQUENCY (Hz)

FIG 2



0.015
0.01
0.005
VOLUME FRACTION (%)
0
0.02
0.04
0.06
0.08
0.1
0.12
RADIUS (mm)

FIG 3


40
35
30
25
20
15
10
5
0
ATTENUATION (dB/cm)
0
50
100
150
200
250
FREQUENCY (kHz)
1
2
3